Combined Free and Forced Convection in a Constant Temperature Horizontal Tube

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An experimental study of superposed free and forced convection for air in a horizontal tube is reported. The laminar Nusselt number, based on the log mean temperature difference, ranged from 29.9 to 15.2; the laminar flow Graetz number, based on the bulk or average temperature of the air, ranged from 33 to 1,300, the Grashof-Prandtl modulus based on properties of air at the wall temperature ranged from 1.1 x 10⁶ to 2.2 x 10⁶. The Grashof number utilized the log mean temperature difference. An analysis of the system from a macroscopic viewpoint led to the determination of an equation which fits the laminar flow experimental data in the range of Graetz numbers from 60 to 1,300. An equation was also found for the turbulent data.

Because the experimental system studied in this investigation involved forced convection in a constant temperature horizontal tube for a range of low Reynolds numbers, the literature pertaining to superposed forced and free convection was of interest. Martinelli and Boelter (*1*) presented the following equation for a system of superposed free and forced convection in a horizontal tube:

$$(N_{Nu})_a = 1.75\, F_1 \sqrt[6]{(N_{Gz})^2 + \left[0.0722\, F_2 \left(\frac{N_{Gr} N_{Pr} D}{L}\right)_w^{0.84}\right]^2} \qquad (1)$$

In Equation (1) the Nusselt number is determined in terms of the arithmetic-mean temperature difference (the arithmetic average of the initial and final temperature differences); the Graetz number is in terms of the mean fluid temperature, and the Grashof-Prandtl-D/L term is based on fluid properties at the wall temperature. The factors F_1 and F_2 (see reference *1*) are functions of $(N_{Nu})_a/N_{Gz}$. In the above equation the Graetz number was squared to account for the horizontal forces influencing heat transfer, and the Grashof-Prandtl-D/L term was squared to take care of the vertical forces influencing the heat transfer. In other words because these forces act at 90 deg. to each other in a horizontal tube the term represented by the radical was considered to be the vectorial sum of two sets of forces.

Eckert and others (*2*) have reported data on either predominantly free or predominantly forced convection and combined free and forced convection for air in a tube of $L/D = 5$. Based on data in the literature they also have suggested criteria for determining the nature of the process (whether primarily free, mixed, or forced convection) in terms of the Graetz number and the Grashof-Prandtl-D/L modulus for laminar flow, the Reynolds number and the Grashof number in the transition regions, and the Reynolds number and the Grashof-Prandtl modulus for turbulent flow.

This paper summarizes experimental work on superposed free and forced convection of air in a low L/D ratio horizontal heated tube. The laminar flow data were in the combined flow region as established by Eckert and others (*2*).

The data reported on here were the calibration runs for a horizontal tube. The primary purpose of the experimental apparatus was to furnish experimental data on the effects of resonant acoustic vibrations on the heat transfer coefficient. The acoustic data have not yet been completed; however the results to date together with the results reported on here are given in tabular form in reference 3.

DESCRIPTION OF APPARATUS

The experimental system is shown schematically in Figure 1. The test section, which consists of a 10 ft. long by 4⅛-in. O.D. copper tube with a ⅛-in. wall, is surrounded by a 16-in. standard pipe to form an annular space. Steam is admitted to the annular space and condenses on the outer surface of the copper tube. The amount of condensate, which is a measure of the heat flow through the tube wall, is collected from around the periphery and along the length of the copper tube by fourteen separate chambers. The condensate collecting chambers are separated by partitions spun from 24-gauge soft copper sheet. The partitions were soldered to the tube at intervals of 6 in. along the first 4 ft. and at intervals of 12 in. for the final 6 ft. of the 10-ft. test section.

At the bottom of each compartment a Parker fitting was provided to permit drainage of the condensate from the chamber. From each fitting a ¼-in. O.D. copper tube passes the condensate through the end flange of the apparatus into a burette. The transfer tube from the end flange to the burette is rubber and is filled with condensate to provide a liquid seal from the steam space to the outside air. Wicks are provided in the collectors, and transfer cups are provided to maintain a continuous drainage of the condensate.

Saturated steam was supplied to the apparatus by a 1-in. diameter steel pipe. The pressure in the steam chest was approximately 6 in. of water. Atmospheric pressure was determined by a mercury

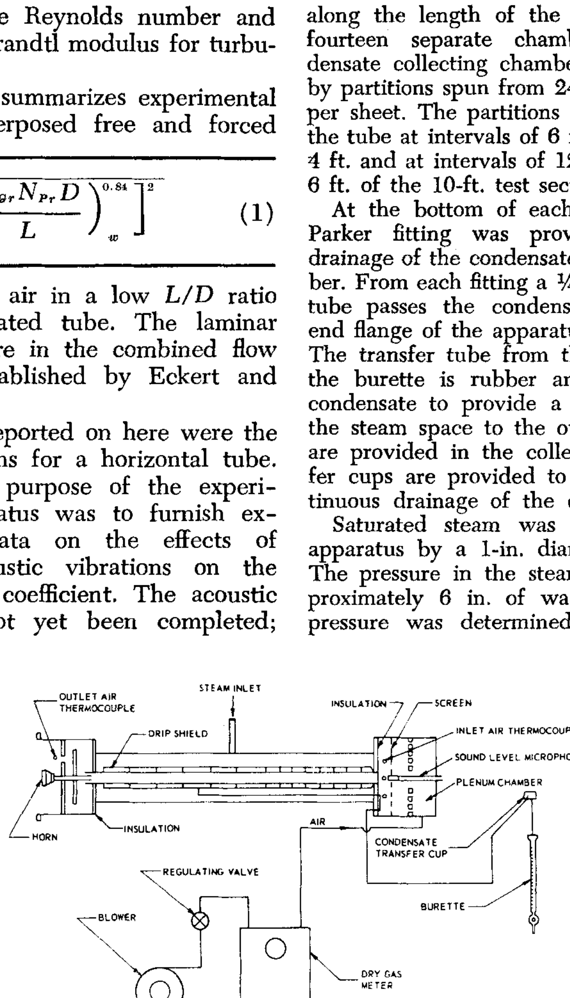

Fig. 1. Schematic drawing of apparatus.

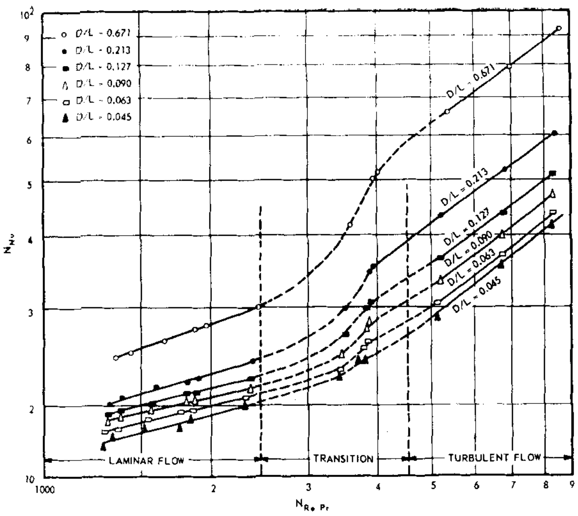

Fig. 2. Nusselt number vs. Reynolds-Prandtl number for no-sound runs.

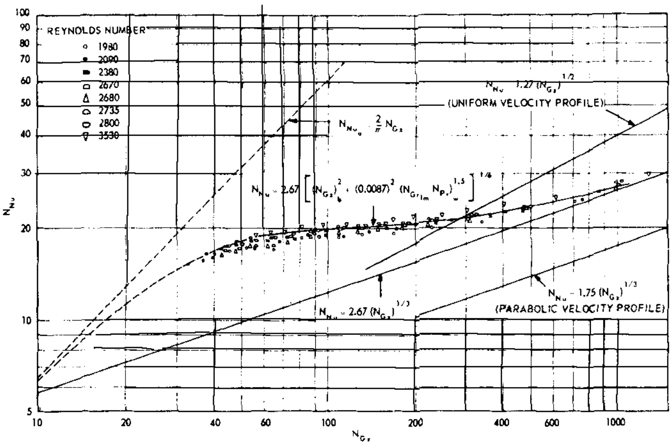

Fig. 3. Nusselt number vs. Graetz number, no-sound experimental data (10 ft. long, 3.875-in. I.D. isothermal tube).

barometer, and the steam temperature was determined from the table of properties of steam (*4*). The temperature of the test-section wall was measured by six 24-gauge copper-constantan thermocouples soldered to the top of the test tube. The leads were taken out of the steam chest through thermocouple fittings. Four thermocouples equally spaced around the air entrance to the test section gave an accurate indication of the inlet air temperature. One thermocouple at the exit of the test section indicated the exit air temperature. Large, air-tight wooden boxes were built for the ends of the test assembly. The boxes are shown in Figure 1 and were used as plenum chambers.

A centrifugal blower, blowing ambient air through a positive displacement type of gas meter into the inlet plenum chamber, furnished the air to the apparatus. From the inlet chamber the air flowed horizontally through a baffle and screen and finally into the test section. The air flow rate was controlled by adjusting the outlet from the centrifugal blower. Flow rate was determined by measuring the time required for a particular volume of air to flow through the meter. Air temperatures were measured at the inlet to the blower and at the entrance and exit of the test section. The entrance and exit thermocouples were made of 30-gauge copper-constantan wire.

The entrance air appeared to be fairly well mixed, and no special mixing device was employed. At the air outlet however a baffle system was provided to insure good mixing prior to temperature measurement. Properties of the air for the conditions of the experiment were taken from reference 5.

In an effort to study the effects of acoustic vibrations on heat transfer to air the heat transfer tube was resonated at various frequencies and sound levels by means of a conventional loud-speaker driver with a 12-in. long horn (see Figure 1). These data have not yet been analyzed completely and will be presented at a later date.

In all the tests reported here the apparatus was allowed to run at least 2 hr. to assure thermal equilibrium before any data were recorded. The data taken for each run were as follows: the water levels in the burettes, the inlet and outlet air temperatures, the air flow rate (measured several times by observing the time-integrated flow rate through the dry gas meter), the barometric pressure, dry-bulb temperature, wet-bulb temperature, and steam pressure values.

A typical test run required 2 hr. or more. Heat balances determined from comparing the temperature rise of the mass air flow rate to the total condensate rate were within ± 5%.

EXPERIMENTAL RESULTS AND DISCUSSION

As mentioned before Martinelli and Boelter (*1*) have suggested that Equation (1) may be used for superposed free and forced convection in a horizontal tube. This equation is for fully developed flow and therefore is not suited for comparison with the undeveloped flow data obtained in this investigation. It does however indicate a method of adding effects which was utilized, at least partially, in the development of the correlating equation for the laminar flow data.

The sharp-edged entrance of the experimental apparatus, while practical in heat transfer equipment, does not lend itself readily to theoretical analysis. Consequently the experimental results were plotted as Nusselt numbers vs. the Reynolds-Prandtl product (Figure 2). The data indicated a transition from laminar flow to transition flow at a Reynolds-Prandtl product of approximately 2,450. Below this value the Nusselt number is a function of the Reynolds-Prandtl product to the 1/3 power. This is indicated by the 1/3 slope of the various *D/L* ratios at the low Reynolds numbers. Above the transition zone, as indicated in Figure 2, the slope is 0.8 which is representative of turbulent flow.

The laminar data from this investigation are plotted in Figure 3. The Nusselt numbers are based on the log-mean temperature difference; however for many investigations the difference between the arithmetic and log-mean values is small.

An examination of the data obtained for low Reynolds numbers (below 3,500) for the first few chambers, where free convection forces are unimportant, indicates that the Nusselt number can be represented by

$$N_{Nu} = 2.67\,(N_{Gz})_b^{1/3} \qquad (2)$$

When one utilizes Equation (2) as the Nusselt number equation for the tube with no free convection effects and a Nusselt number equation for free convection laminar flow on a flat plate as the free convection term

$$N_{Nu} = 0.55\,[(N_{Gr})_{lm}\,(N_{Pr})]^{1/4} \quad (3)$$

an equation is developed which fits the experimental data exceedingly well in a range of Graetz numbers from approximately 60 to 1,300. This equation is

$$N_{Nu} = 2.67\,[(N_{Gz})_b^{2} + (0.0087)^2\{(N_{Gr})_{lm}\,N_{Pr}\}_w^{1.5}]^{1/6} \quad (4)$$

and is drawn on Figure 3 utilizing for the Grashof-Prandtl number terms the data for the 2,800 Reynolds number run. The scatter of the data on Figure 3 is due to the influence of the Grashof-Prandtl numbers. Because the inlet air temperature varied from day to day the experimental Grashof-Prandtl numbers also varied and this contributed to the slight scatter as indicated in the figure.

The data of this investigation are higher than those indicated by the well-known Sieder-Tate equation. The equipment, as indicated in Figure 1, did not have an entrance calming section. The fluid therefore entered the tube, and in the first few feet a boundary layer built up essentially as on a flat plate. When one utilizes the formula for the development of a boundary layer on a flat plate, it is apparent that for the tube lengths under consideration, with air as the heat transfer medium, the boundary layer did not fill the tube. The data therefore in the first few feet closely approaches that indicated by Kays (*6*).

Also indicated on Figure 3 are the curves and equations for uniform and parabolic velocity profile flow. As would be expected the experimental data of this investigation without free convection effects fall between these two curves. The flow in the experimental apparatus is neither parabolic nor uniform; it is a combination of both, since the boundary layer does not fill the tube.

Below a Graetz number of 60 the experimental data deviated from Equation (4). This is to be expected because the initial correlation used for forced flow, Equation (2), is not valid at low Graetz numbers. Below a Graetz number of 60 the experimental data of this investigation tend towards the curve represented by

$$(N_{Nu})_a = \frac{2}{\pi} N_{Gz}$$

This equation is for the special limiting case where the fluid is heated nearly to the constant temperature of the tube wall (*7*).

In Equation (4) the Grashof number is based upon the log-mean temperature difference. Changing the log-mean temperature difference in the Grashof number to a temperature difference based upon the inlet air and wall temperatures is comparatively straightforward. This analysis leads to the following equation for the Nusselt number in the laminar range where superposed free and forced convection exists:

$$N_{Nu} = 2.67 (N_{Gz})_b^{1/3} \left\{ 1 + \frac{(0.0087)^2}{(N_{Gz})_b^{1/2}} \left[\frac{N_{Gr} N_{Pr}}{\pi N_{Nu}} \left(1 - e^{-\frac{\pi N_{Nu}}{(N_{Gz})_b}} \right) \right]^{1.5} \right\}^{1/6} \quad (5)$$

It can be seen from Equation (5) that the calculation of a heat transfer coefficient is a trial-and-error procedure. First a Nusselt number is assumed, and the equation is used to calculate it. If the proper calculated value agrees with the one which is assumed, then the proper Nusselt number is obtained.

Figure 2 indicates that above Reynolds numbers of approximately 3,500 a transition to turbulent flow takes place. For Reynolds numbers above 7,000 the experimental data were correlated within ± 5% by the following equation:

$$N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^{1/3} [1 + 3\,D/L] \quad (6)$$

This correlation is valid for all values except those for the first two or three chambers or the first 1 ft. of the tube. Free convection forces are negligible in the turbulent flow region.

DERIVATION OF EQUATION (4)

For the experiments discussed herein

$$N_{Nu} = 2.67 (N_{Gz})_b^{1/3} \quad (7)$$

is the formula which was used to represent a laminar heat transfer coefficient for forced convection. Equation (7) does not consider free convection effects.

From reference *8* for free convection on a vertical wall

$$\frac{hL}{k} = 0.548 \left[\left(\frac{c_p\mu}{k} \right) \left(\frac{L^3 \beta g (T-t)}{\nu^2} \right) \right]^{1/4} \quad (8)$$

If the diameter of the tube can be considered to be equal to the height in Equation (8), then

$$N_{Nu} = 0.55 (N_{Gr} N_{Pr})^{1/4} \quad (9)$$

In Equation (9), as applied to the horizontal tube, the temperature difference causing the free convection can be considered to be the log-mean temperature difference from the entrance to the point under consideration.

When one utilizes the method suggested briefly in reference *9* it is first necessary to determine a relationship between the forced and free convection forces. This is done here by equating Equations (7) and (9):

$$N_{Nu} = 2.67 (N_{Gz})_b^{1/3} = 0.55 \{(N_{Gr})_{lm} N_{Pr}\}_w^{1/4}$$

From the above

$$(N_{Gz})_b = \left(\frac{0.55}{2.67}\right)^3 \{(N_{Gr})_{lm} N_{Pr}\}_w^{3/4} = 0.0087 \{(N_{Gr})_{lm} N_{Pr}\}_w^{3/4} \quad (10)$$

Equation (10) therefore represents an equivalent Graetz number for free convection.

By adding the free and forced convection flows vectorially, as suggested by Martinelli and Boelter (*1*), and utilizing the forced convection Graetz number and the free convection equivalent Graetz number as determined above, one obtains

$$N_{Nu} = 2.67 [(N_{Gz})_b^2 + (0.0087)^2 \{(N_{Gr})_{lm} N_{Pr}\}_w^{1.5}]^{1/6} \quad (11)$$

which is Equation (4). In Equation (11) the Grashof number is based upon the log-mean temperature difference. As indicated in reference 9 the log-mean temperature difference for a tube is

$$\frac{\Delta t_{lm}}{T - t_o} = \frac{N_{Re} N_{Pr} \frac{D}{L}}{4 N_{Nu}} \left(1 - e^{-\frac{4 N_{Nu}}{N_{Re} N_{Pr} D/L}} \right) \quad (12)$$

Consequently, when one utilizes Equation (12), Equation (11) becomes

$$N_{Nu} = 2.67 (N_{Gz})_b^{1/3} \left\{ 1 + \frac{(0.0087)^2}{(N_{Gz})_b^{1/2}} \left[\frac{N_{Gr} N_{Pr}}{\pi N_{Nu}} \left(1 - e^{-\frac{\pi N_{Nu}}{(N_{Gz})_b}} \right) \right]^{1.5} \right\}^{1/6} \quad (13)$$

because $N_{Re} N_{Pr} D/L = 4/\pi\, N_{Gz}$, and the Grashof number is now based upon $(T - t_o)$.

CONCLUSIONS

A system having superposed free and forced convection of air in a horizontal tube was examined, and equations for the Nusselt number were derived which fit the experimental data for both laminar and turbulent flow conditions. The data could not be compared with the Martinelli-Boelter and Sieder-Tate equations because of the difference in entrance conditions.

ACKNOWLEDGMENT

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NOTATION

N_{Gz}	=	Graetz, $(Wc_p)/kL$), dimensionless
N_{Gr}	=	Grashof, $gD^3\beta(T - t_o)/\nu^2$, dimensionless
N_{Nu}	=	Nusselt, hD/k, dimensionless
N_{Pr}	=	Prandtl, $c_p\mu/k$, dimensionless
N_{Re}	=	Reynolds, VD/ν, dimensionless
D	=	diameter of tube, ft.
F_1, F_2	=	see reference *1*

L = test section length, ft.
T = wall temperature, °F.
V = mean velocity, ft./sec.
W = mass flow rate, lb./hr.
c_p = heat capacity at constant pressure, B.t.u./lb. °F.
g = gravitational acceleration, ft./sec.^{2}
h = heat transfer coefficient, B.t.u./hr. sq. ft. °F.
k = thermal conductivity, B.t.u./hr. ft. °F.
t = fluid temperature, °F.

Greek Letters

β = coefficient of volumetric expansion, °F.$^{-1}$
μ = dynamic viscosity, lb./ft. hr.
ν = kinematic viscosity, sq. ft./sec.
ρ = density, lb./cu. ft.
Δ = difference (that is Δt is temperature difference)

Subscripts

a = based on arithmetic average temperature difference
b = based on bulk fluid temperature
lm = based on logarithmic-mean-temperature difference
L = condition at distance L from entrance
o = condition at entrance
w = based on tube wall temperature

LITERATURE CITED

1. Martinelli, R. C., and L. M. K. Boelter, "University of California Publications in Engineering," Vol. 5, No. 2, pp. 23-58 (1942).
2. Eckert, E. R. G., and A. J. Diaguila, *Trans. Am. Soc. Mech. Engrs.*, **76**, 497 (1954).
3. Spurlock, Jack M., Thomas W. Jackson, Kenneth R. Purdy, Calvin C. Oliver, and Harold L. Johnson, *WADC Technical Note* **59-330** (June 1959).
4. Keenan, J. H., and F. G. Keyes, "Thermodynamic Properties of Steam," Wiley, New York (1948).
5. McAdams, W. H., "Heat Transmission," 3 ed., p. 483, McGraw-Hill, New York (1954).
6. Kays, W. M., *Trans. Am. Soc. Mech. Engrs.*, **66**, 1265 (1955).
7. McAdams, W. H., "Heat Transmission," 3 ed., p. 232, McGraw-Hill, New York (1954).
8. *Ibid.*, p. 170.
9. Jackson, T. W., W. B. Harrison, and W. C. Boteler, *Trans. Am. Soc. Mech. Engrs.*, **80**, 739 (1958).

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Turbulent Newtonian Flow in Annuli

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In recent years there have been quite a few experimental studies on turbulent flow in annuli. In this paper a Prandtl mixing-length approach is applied to give a friction factor vs. Reynolds number expression for annuli [see Equation (22) and Table 1]; this expression describes tube flow and slit flow as special cases. No new adjustable constants appear in the final result other than those determined earlier for tube flow. The final expression is found to predict friction factors within the accuracy of the existing experimental data. The mixing-length friction-factor expression is thus substantially more accurate than the usual hydraulic-radius procedure and of comparable accuracy to other recent annulus friction-factor treatments.

Simple empirical theories of turbulence have often been used to describe the relation between pressure drop and flow rate for the turbulent flow of Newtonian fluids in smooth tubes. In this paper the authors show how one of these theories, the Prandtl mixing-length model, can be applied quite successfully to the turbulent flow of Newtonian fluids in annuli.

DEFINITION OF FRICTION FACTOR AND REYNOLDS NUMBER

Pressure-drop data for the flow of Newtonian fluids in smooth concentric annuli can be correlated by the use of three dimensionless groups. In general these groups will be a friction factor, a Reynolds number, and a geometric factor a.

A general definition for the friction factor is (*1*)

$$F = A K f \tag{1}$$

For an annulus the total drag force on the entire wetted surface is $\Delta p \pi R^2 (1 - a^2)$. The characteristic area may be taken as the total wetted area $2\pi RL (1 + a)$. The characteristic kinetic energy per unit volume may be chosen to be $\frac{1}{2}\rho V^2$. Thus for annuli the friction factor is defined by

$$f = \frac{R(1-a)}{L} \frac{\Delta p}{\rho V^2} \tag{2}$$

This is the same friction factor which is obtained by use of the mean-hydraulic-radius and which is commonly used for annuli (*9, 12*).

For the isothermal, steady, axial flow of an incompressible fluid in a coaxial annulus integration of the equation of motion gives the following well-known expression for the flux of z-momentum in the r-direction (*1*, Equation 2.4-3):

$$\tau = \frac{R \Delta p}{2L} \left(\frac{r}{R} - \lambda^2 \frac{R}{r} \right) \tag{3}$$

This momentum-flux distribution is shown in Figure 1, where one can see that the momentum flux is negative at the inner wall ($r = aR$), that it becomes zero at some intermediate radial distance ($r = \lambda R$), and that it is positive at the outer wall ($r = R$). The quantity λ is defined as that value of $\xi = r/R$ for which the local velocity is a maximum and the momentum flux is zero.

For the laminar flow of Newtonian fluids $\tau = -\mu\,(du/dr)$; insertion of this expression into Equation (3) and integration gives the velocity distribution:

$$u = \frac{R^2 \Delta p}{4 \mu L} \left[1 - \xi^2 + \frac{1 - a^2}{\ln (1/a)} \ln \xi \right] \tag{4}$$